BETA-ADJUSTED COVARIANCE ESTIMATION

Kris BOUDT (UGent, VUB, VU), Kirill DRAGUN (VUB), Orimar SAURI(AAU), Steven VANDUFFEL (VUB)

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Objectives

Several Exchange Traded Funds (ETFs) are more liquid than many of their component stocks.

- **Exploit extra source of information in covariation of stocks with ETF to improve pre-estimators of the realized covariance of the stocks included in the ETF.**
- **Assess performance of BAC and compare it with pre-estimators**

Main Result

– a continuous Itô's semimartingale, $X_t = X_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dB_s,\ \ t \geq 0$,
– the object of interest is the integrated covariance matrix of the process X_t over the interval $[0,1] : \Theta = \int_0^1 \Sigma_s ds$.
– the corresponding log-transformed Net Asset Value (NAV) is equal to the natural logarithm of the weighted sum of the component prices of the ETF: $Y_t^* = \log\left(\sum_{k=1}^d a_t^k \exp(X_t^k)\right)$.
– we define the stock-ETF β^l associated to the l-th asset as the quadratic covariation between X^l and $\exp(Y^*)$. It follows: $\beta^l = \sum_{k=1}^d \int_0^1 w_s^k \Sigma_s^{kl} ds$, where $w_s^l = a_s^l \exp(X_s^l)$.
– ***the proposed BAC estimator of the integrated covariance* is then:** $\overline{\Sigma}^{BAC} = \overline{\Sigma} - \overline{\Delta}^{BAC}$, **with** $\mathrm{vec}(\overline{\Delta}^{BAC}) = L(\overline{\beta} - \beta\bullet)$.
– and $L := \left(I_{d^2} - \frac{1}{2}\mathcal{Q}\right)\bar{W}'\left(I_{d^2}\left(\sum_{k=1}^d \frac{\sum_{m=1}^{n_k}(w_{t_{m-1}^k}^k)^2}{n_k}\right) - \frac{\bar{W}\mathcal{Q}\bar{W}'}{2}\right)^{-1}$.

Beta framework, properties and extensions

- We consider a $d \times d$ adjustment process Δ_s and define $\overline{\beta}_\Delta^l = \sum_{k=1}^d \sum_{m=1}^{n_k} w_{t_{m-1}^k}^k \left(\hat{\Sigma}_{t_{m-1}^k}^{lk}(t_m^k - t_{m-1}^k) - \Delta_{t_{m-1}^k}^{lk}\right)$.
- We impose equality between stock-ETF beta and target beta under the criterion of minimizing the distance between the adjusted estimator and the pre-estimator $\overline{\beta} - \overline{\beta}_\Delta = W\delta$, where $\delta = (\delta^{11\prime}, \delta^{12\prime}, \ldots, \delta^{1d\prime}, \ldots, \delta^{d1\prime}, \ldots, \delta^{dd\prime})'$, $\delta^{lk} = (\Delta_{t_0^k}^{lk}, \ldots, \Delta_{t_{n_k-1}^k}^{lk})'$, W be a $d \times dn$ matrix whose ith row is given by
$$W^i = (0'_{(i-1)n}, w', 0'_{(d-i)n}),$$
leading to the following optimization problem
$$\hat{\delta} = \underset{\delta}{\operatorname{argmin}}\, \delta' P \delta, \quad \text{s.t.} \quad \begin{cases} W\delta = \overline{\beta} - \beta_\bullet; \\ Q\delta = 0_{(d-1)d/2}, \end{cases}$$
reflecting beta and symmetry constraints.
- We derive the asymptotic distribution of the BAC adjustment for Hayashi-Yoshida estimator. Under certain assumptions $\sqrt{n}\left(\overline{\beta} - \beta\right) \xrightarrow{s.d} MN(0, \Psi)$, where
$$\Psi = \int_0^1 \Sigma_s(w_s' \Sigma_s w_s') ds + \int_0^1 \left[(\Sigma_s w_s)(\Sigma_s w_s)'\right] ds.$$
- First extension is to mitigate the influence of market microstructure effects and to deal with price jumps by comparing them with a multiple κ of an estimate of the local variance,
$$F_k = \bigcup_i (t_{i-1}^k, t_i^k] \quad s.t. \quad (\tilde{X}_{t_i^k}^k - \tilde{X}_{t_{i-1}^k}^k)^2 > \kappa s_{t_i}^k + 2\sigma_\zeta^k, \qquad F = \bigcup_k F_k,$$
where $s_{t_i}^k$ is a jump robust estimate of the quadratic variation of X_t on the interval $(t_{i-1}^k; t_i^k]$.
- Our second improvement is to exploit the information in the realized variance of the ETF prices
$$\frac{\overline{\gamma} - \gamma_\bullet}{\sum_{j=1}^d n_j^{-1} \sum_{m=1}^{n_j}(w_{t_{m-1}^j}^j / \exp(2Y_{t_{m-1}^j}))^2} \left(n_1^{-1}\sum_{m=1}^{n_1} \frac{w_{t_{m-1}^1}^1}{\exp(2Y_{t_{m-1}^1})}, \ldots, n_d^{-1}\sum_{m=1}^{n_d} \frac{w_{t_{m-1}^d}^d}{\exp(2Y_{t_{m-1}^d})}\right)'.$$

Simulation results

We conduct a Monte Carlo study to evaluate the accuracy gains:

- over 50% in the case in which the oracle beta is used as target,
- for traditional realized covariance (RC) up to 74%,
- up to 20% when target beta is estimated and for HY estimator

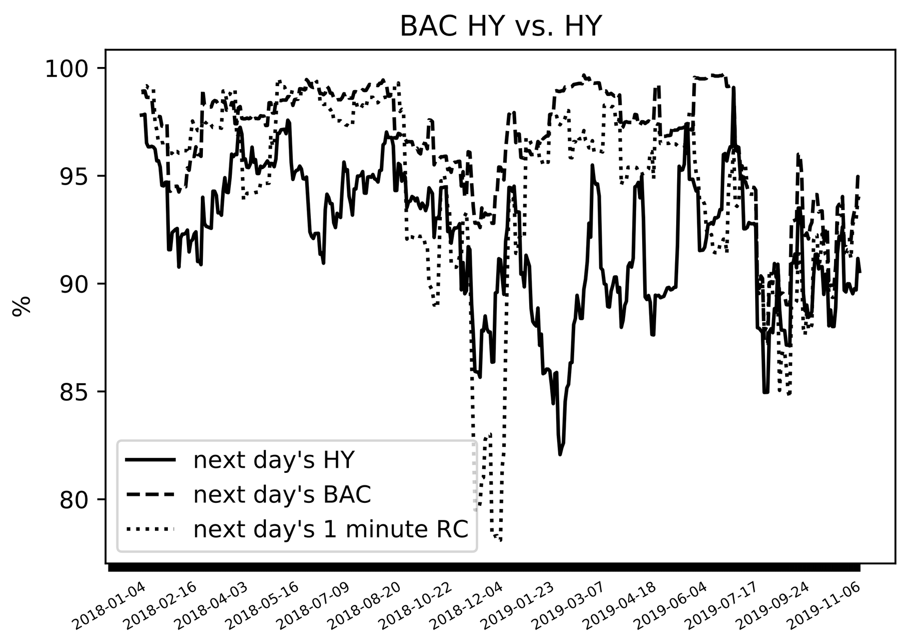
Table 1: Mean squared error of integrated covariance estimates ($\kappa = 0.085$)

	Pre-estimator	Initial $\overline{\Sigma}$ no jumps	BAC with $\beta_\bullet =$ β	$\overline{\beta}^Y$	$\overline{\beta}^{VAB}$	Initial $\overline{\Sigma}$ with jumps	BAC with $\beta_\bullet =$ β	$\overline{\beta}^Y$	$\overline{\beta}^{VAB}$
Panel A: No noise									
10	HY	0.00023	0.00008	0.00021	0.00018	0.00024	0.00008	0.00022	0.00019
	RC	0.04165	0.01161	0.01177	0.01173	0.04223	0.01167	0.01185	0.01180
	TSC	0.00238	0.00060	0.00075	0.00071	0.00224	0.00055	0.00070	0.00066
30	HY	0.00026	0.00010	0.00023	0.00020	0.00025	0.00009	0.00023	0.00020
	RC	0.05437	0.01282	0.01296	0.01293	0.05441	0.01306	0.01330	0.01325
	TSC	0.00265	0.00068	0.00083	0.00080	0.00257	0.00063	0.00081	0.00076
100	HY	0.00025	0.00011	0.00023	0.00021	0.00028	0.00012	0.00025	0.00023
	RC	0.05954	0.01544	0.01555	0.01552	0.06088	0.01567	0.01595	0.01588
	TSC	0.00277	0.00080	0.00094	0.00090	0.00262	0.00077	0.00094	0.00090
Panel B: Noise									
10	HY	0.00028	0.00010	0.00025	0.00022	0.00027	0.00010	0.00023	0.00021
	RC	0.04177	0.01155	0.01165	0.01164	0.04177	0.01170	0.01183	0.01181
	TSC	0.00236	0.00060	0.00076	0.00073	0.00240	0.00058	0.00073	0.00070
30	HY	0.00029	0.00012	0.00026	0.00024	0.00028	0.00011	0.00025	0.00023
	RC	0.05394	0.01292	0.01306	0.01304	0.05368	0.01266	0.01285	0.01282
	TSC	0.00270	0.00070	0.00086	0.00083	0.00279	0.00073	0.00089	0.00086
100	HY	0.00028	0.00013	0.00026	0.00024	0.00030	0.00013	0.00027	0.00025
	RC	0.05950	0.01562	0.01574	0.01572	0.06089	0.01556	0.01591	0.01585
	TSC	0.00253	0.00076	0.00090	0.00088	0.00296	0.00085	0.00104	0.00101

Index tracking portfolio

Now we want to evaluate BAC performance on market data via its application to index tracking. The next days's tracking error is:
$$TE_{t+1}(\alpha(\hat{\Omega}_t)) = \hat{\omega}_{Y,t+1} - 2\hat{\omega}_{YK,t}\hat{\Theta}_{K,t}^{-1}\hat{\omega}_{YK,t+1} + \hat{\omega}_{YK,t}\hat{\Theta}_{K,t}^{-1}\hat{\Theta}_{K,t+1}\hat{\Theta}_{K,t}^{-1}\hat{\omega}_{YK,t}.$$

Figure 1: Percent of outcomes where the BAC estimator-based tracking portfolio has a lower next-day's tracking error (evaluated using HY or 1-min RC) than the pre-estimator



Overview

- We improve a realized covariance estimate by imposing the condition that the covariance-implied stock-ETF beta equals the direct stock-ETF beta estimate.
- We obtain the asymptotic distribution for the BAC estimator when the HY estimator is used as a a pre-estimator.
- In the simulation study, we show substantial gains for BAC vs. pre-estimators. These results are robust to the presence of microstructure noise and jumps.
- In empirical application the problem of minimization of tracking error of the hedged portfolio using BAC is solved more efficiently compared to initial estimates.